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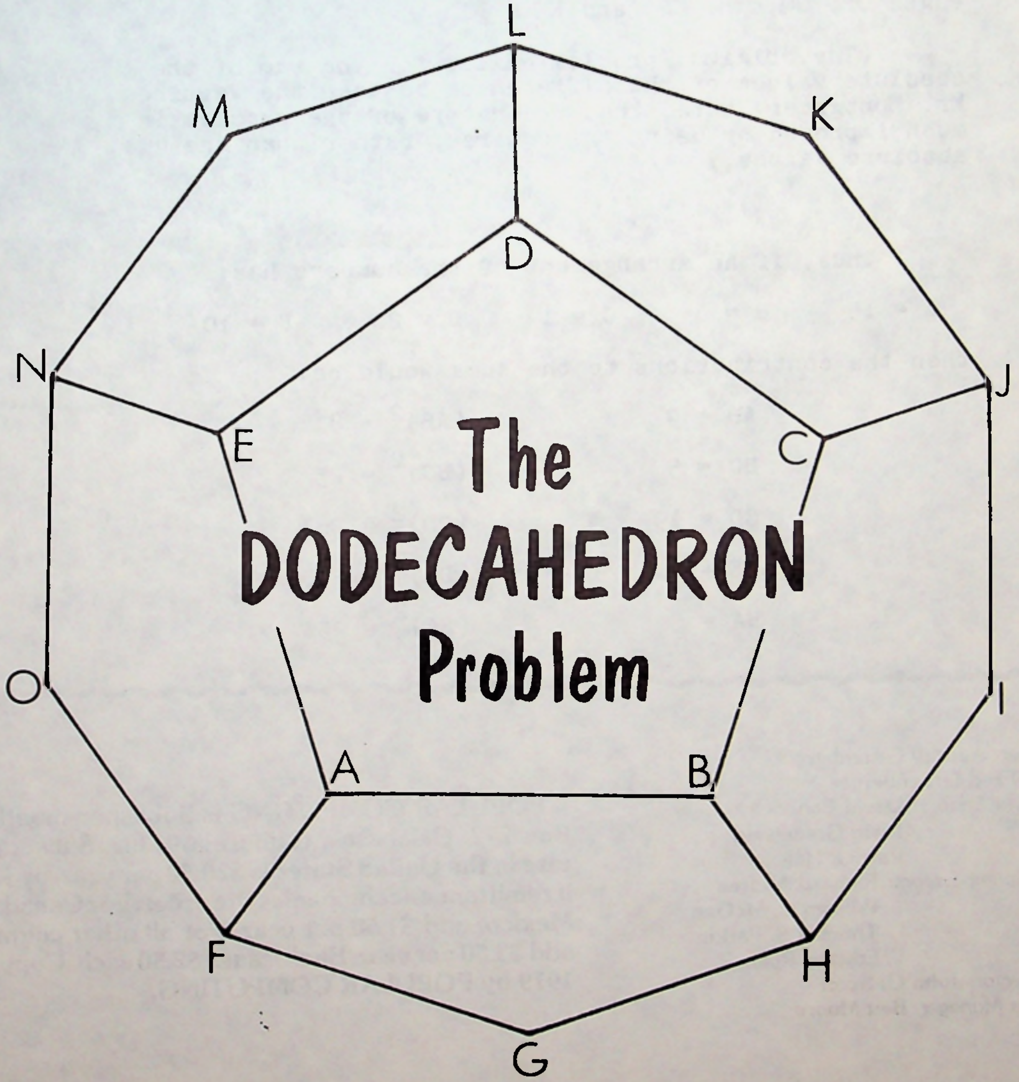
The DODECAHEDRON Problem



The Dodecahedron Problem

A solution to Problem 226 (issue number 61) is furnished by Ralph Montgomery, of St. John's Community College.

The problem was this: to number the 20 vertices of a dodecahedron with the numbers from 1 to 20 in such an order that the sum of the squares of the 30 differences along the edges would be (a) a maximum, or (b) a minimum.

In order to communicate a solution to this problem, it is necessary to establish an ordering to the 20 vertices. Fifteen vertices can be ordered as shown in the accompanying diagram; the remaining 5 are ordered so that there are edges OP, GQ, IR, KS, and MT.

(The original problem called for the sum of the absolute values of the differences between the vertices. Mr. Montgomery noted that the nature of the problem is much improved by using the squares, rather than the absolute values.)

Thus, if an arrangement of the numbers has:

A = 15 B = 6 C = 1 D = 20 E = 10

then the contributions to the sums would be:

AB = 9	$(AB)^2 = 81$
BC = 5	$(BC)^2 = 25$
CD = 19	$(CD)^2 = 361$
DE = 10	$(DE)^2 = 100$
EA = 5	$(EA)^2 = 25$

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Mr. Montgomery's solution began by forming a random permutation of the numbers from 1 to 20. Two arrays, A and B, were set up. All elements of array A were set to zero, and for array B, B(I) = I. A random number between 1 and 20 was drawn. If that number existed in array B, it was placed in the next available position in array A, and blanked out to zero in array B. If it did not exist in B, a new random number was drawn. This procedure was repeated until 18 numbers were furnished to array A; then the remaining two non-zero numbers in array B were moved to A(19) and A(20).

For each random permutation generated by the above scheme, the sum of squares of the dodecahedron edge numbers was calculated. If this sum was larger than any previous sum, or was smaller than any previous sum, the program listed the permutation.

The program was run for several thousand permutations, with the results shown in the Table. It is unlikely that the results shown are the best possible, but it is even less likely that the best results would differ from these by 1%.

Vertex	Contents for MIN	Contents for MAX
A	7	13
B	9	6
C	20	1
D	19	14
E	13	5
F	8	12
G	6	8
H	12	20
I	11	4
J	18	19
K	16	2
L	10	11
M	15	7
N	14	17
O	17	10
P	2	16
Q	1	9
R	3	15
S	5	18
T	4	3
Sum of squares:	1110	3160

Numbers

by Norman Sanders

For nigh on thirty years now my greatest kick has been producing numbers on a printer attached to a computer. If it hadn't been for the computer I'd have had to work for a living. Instead I've been paid for the pleasure of enjoying myself. But there's a deep mystery at hand, and I'm not alone in not fathoming it. There are hundreds of thousands of other people who also presumably experience it, and without it the computing revolution wouldn't have happened. So I think it's worthwhile trying to get at the heart of this matter.

A computer is a lot of different things to a lot of very different people, and the satisfaction they derive from their participation must be very different. Getting the maze of circuitry actually working, especially in the crocodile clip era, must be a heady experience for the designer. Brave Cortez on a peak in Darien, no doubt. Seeing your risky shares blossom into a modest fortune isn't without its emotional overtones. Leading a project to successful fruition without loss of hair or head is a highly satisfactory achievement. But nothing equals the tingling in the spine of a page of accurate numbers after the 99th attempt: the core of what computing is all about.

But does it matter what the numbers signify? To some obviously it does. A geologist doesn't get terribly worked up about accounts receivable. But I don't think I'm alone in experiencing equal joy from whatever application. I've produced random numbers (deliberately), primes, integrals, cutter centre paths, school timetables, probabilities and cost accounts. That's because I'm not a geologist. (Or vice-versa.) I don't have a profession. If pressed, to cut off the discussion I say that computing is my profession. That statement has no meaning, of course, but it sounds valid.

So my enjoyment is number independent. And that has a great advantage of increasing my employability. I'll go anywhere and do anything for numbers. I produced my first numbers on a hand-cranked Brunsviga (none of your pocket calculators for me thank you) and at one time thought I'd beat Hartree's 10,000 hours of high-speed wrist work. But that was not to be. Someone started making automatic computers that actually worked. They even put them inside metal cabinets. (I mean how much are you going to let your sales department interfere? There must be a limit.) So the production of numbers took on a radically different form. One became removed from the actual cranking, and fell back to working with the logic of the thing. And it was then that enjoyment gave way to heady excitement, and mystery entered the picture.

The numbers you produce on a typewriter are more neatly-printed than those from a high-speed printer. And your answers were more accurate on a 10-digit Brunsviga than on a 36-bit computer using floating point. So it wasn't either presentation or precision that gave you the thrill. Was it then the quantity of numbers that you could get a programmed loop to produce? Certainly you'll never produce six million primes on a Brunsviga. But I get just as much of a kick out of a few selected specimens, and I well remember a significant breakthrough based on a single number, 860. It was what went into producing that number that counted.

No, it was something much deeper. It was something to do with logic. But we've had logic since Aristotle, and computer logic since Boole. Moreover, there was nothing preventing algorithmic languages being created in the nineteenth century. Indeed with Algol as numerical partner to Latin, the international exchange of scientific ideas might have experienced a substantial boost from the time of the foundation of the Royal Society.

But there was still something missing. You could have had a magnificent time arguing the relative merits of rival algorithmic languages in the eighteenth century, but you still lacked proof. There's no automatic way of testing an algorithm for correctness. Indeed, what constitutes mathematical proof? It looks all right, but is it?

It isn't until you start getting numbers that you get proof. The computer does what you tell it, not what you think you tell it. And with proof, the excitement that your ideas were right. Somehow the soul of the thing is having your logic carried out for you automatically, the numbers being the bearers of information concerning its correctness.

But even this isn't all. By the end of the fifties I made the amazing discovery that for many people the numbers in printed form weren't really what they wanted. They were indigestable and had to be plotted. So after a few seconds of number production, people would spend weeks transferring the numbers to graph paper. Why not plot them directly? And so I connected a plotter to a computer and wrote one of the first plot routines. This resulted in another quantum jump in sheer exhilaration. To see that arm racing up and down the paper producing beautiful lines at so many inches per second was sheer ambrosia.

So it was then clear that closer to the heart of the matter was the conversion of pure idea into moving machinery. You thought it and it happened! A twentieth century alchemy. Really heady stuff. But even that wasn't the full answer. If you could make mechanical movements in two dimensions and produce lines, you could do so in three dimensions and produce things. Of all the computer applications that I've had anything to do with, the numerical control of machine tools has been that most fraught with awe and wonder. And when you know the details, part of the wonder is that it ever works. There are so many opportunities for getting it wrong, yet it's become part of the solid panoply of the manufacturing industry.

Logic + electronics + mechanical movement = a three-dimensional object. The most exciting product of the twentieth century; equivalent to

$$e^{\pi i} + 1 = 0$$

in its apparent simplicity. But a lot of people would pour scourn upon the idea, as they would computer chess or computer composition of music. The object lacks artistry. And there's another clue there. For myself, I am utterly unable to draw, paint, model, or carve. I am a failed would-be craftsman. Had it not been for the computer I would have been a hewer of wood or a fetcher of water or a stockbroker. Could it be that this is also true of a substantial number of the computing profession? The computer as a replacement for the fingers in carrying out our ideas? The driving force in getting all those projects up and running a substitute for artistic endeavour? What would have happened to Michaelangelo's visions had he been equipped with my motor-nervous system? And how many other village Michaelangelos are there in this world? Is the computer the open-sesame for the frustrations of a myriad immaculate conceptions that would otherwise have died? Is the computer a by-pass mechanism from brain to creation, substituting for the arm and hand? And the whole thing is numerical, albeit disguised in many forms. So is this why I love numbers so?

The fact of the matter is that without this sheer joy of numbers a substantial amount of the best work that has been done lo these thirty years on computers would not have been. Behind each success lies untold-of frustration that salary or managerial directive alone could never have overcome. Only the determination born of spontaneous dedication has made it all possible, as with all other matters of quality in this world, ballet, painting, music, sculpture, writing, whatever.

This may all be true, but there may yet be a solid core to this enigma, that number has a beauty all of its own, unencumbered by association with any application. A number with the beauty of a rose, for those who take the trouble to look.

Norman Sanders is the author of two marvelous books:

The Corporate Computer; How to Live With an Ecological Intrusion (1973)

A Manager's Guide to Profitable Computers (1979)

The essay on "Numbers" goes a long way toward explaining the fascination of computing, which is something that has needed explaining for three decades. (Joe Weizenbaum took a stab at it in his book Computing Power and Human Reasoning, to try to explain compulsive programming.)

What makes Norman's contribution particularly timely is the new phenomenon of professional computer people buying personal computers in large numbers.

Allow me to get personal. For all my working life, back to 1943, I have been surrounded by the latest and best computing machinery, to which I had virtually unlimited access for any useful project (with "useful" defined by me). For many years, I had a choice of several machines, and many languages. For much of the time, there have been competent students around who were eager to program and run anything that I found of interest.

And yet, the tingling that Norman describes is renewed and amplified with the acquisition of a personal machine. This is difficult to explain to non-computer people. The phenomenon of the "busman's holiday" is well known, but there is no common analogy to what we now see; namely, people pursuing as a hobby that which they spend their working hours on as a profession or vocation. It could be that the tingling affects only one in a million people (i.e., Norman and me) and that trying to explain it is akin to a scuba diver trying to explain his hobby to someone who is afraid of the water.

Then again, maybe he has helped to explain it. He is certainly well qualified to try.

Fred Gruenberger

"We've Always Had Inflation"

Discussions of inflation are everywhere. The rate of inflation in the United States is now 13.4 (TIME, July 16), or perhaps at 12.8...or 12.1...it seems to change hourly. Part of the fluctuation that we observe is due to some confusion about the extended effect of short term data. Thus, newspaper articles and TV announcers routinely report a monthly increase of, say, 1.1% and then invariably add that that means an annual rate of 13.2%, apparently on the grounds that twelve times 1.1 is 13.2.

Not so; the laws of compound interest have not been repealed. A monthly increase of 1.1% is equivalent to an annual rate of:

$$(1 + .011)^{12} = 1 + .140286$$

that is, 14.0286%. A monthly rate of 1.1% would amount to a doubling in about 5 1/4 years.

The time of doubling under compound interest can be roughly approximated by the "Rule of 72" which says that the annual interest rate in percent times the time to double in years is 72. The rule assumes annual compounding. A table is given to show the product of the annual interest rate and the time of doubling under various conditions.

An inflation rate of 13.4 would appear as extreme stability to those who have had much worse; for example:

Year	Country	Rate
1979	Brazil	46
1974	Uraguay	107
1973	Chile	508
1975	Chile	340
1978	Chile	30
1976	Argentina	347
1978	Argentina	170
(Source:	London Observer News Service)	
1978	Peru	40

It is extremely difficult to measure inflation with any precision. A car that once cost $2000 is now $6000--but it's not the same car by any means; today's cars have all sorts of mandated gadgetry that wasn't even known a decade ago. So it goes with houses, clothes--even food. We constantly find ourselves trying to compare things that don't compare.

In the table with this article, for example, we show the effects of inflation on the price of a 200-sheet box of Kleenex. But the indicated rate is not correct, because the Kleenex is not the same. The size and weight of each sheet has been reduced over the years. Even such a standard item as a quart of homogenized vitamin D milk might change over the years, as grading standards for quality go down (there is no known case of quality standards getting stricter).

It doesn't even help to try to relate prices to something else, like man-hours of work. How long did a man have to work, 100 years ago, to earn enough to buy a suit? Even if you could pinpoint that, it still doesn't compare; today's suits are distinctly different (in some ways distinctly better). What vocation has remained stable for even a decade? Is ten hours of police duty in 1929 equivalent to ten hours of police duty today?

Try the same exercise in more recent times with houses. We see a $20,000 house becoming a $100,000 house in, say, 20 years. Just on those figures, the same analysis shows:

$$(1 + .R)^{20} = 100000/20000 = 5$$

$$R = 8.38\%$$

which is a fair approximation to the inflation rate of recent years.

Nevertheless, if "we've always had inflation," it must have been quite mild at one time. Let's try a rough analysis. Suppose that in 1889 an average worker worked half an hour to earn a loaf of bread which cost 10¢ at that time. Today the same average worker works 10 minutes to earn a loaf of bread, which now costs 85¢. In one sense, the price has actually gone down. In another sense, we have a similar (but not identical) item going from 10¢ to 85¢ in 90 years. What is the inflation rate on that item? We have:

$$(1 + R)^{90} = 85/10$$

which yields a value of R of 2.4%. Thus, on things that can be calculated (albeit crudely), we seem to have had a long-term rate of inflation that was quite low. (At 2.4%, money doubles in 29.2 years, with annual compounding.) The term "inflation" wasn't even used for rates that low.

Item	Inflation rate, %	From --to	Was then	Is, or will be
Salary	13.4	1979-2000	$15000	$210,361
Shoes	13.4	1979-2000	50	701
Car	7.5	1946-1979	1000	10,876
Car	7.5	[1979-1946]	735.5	8,000
Car	13.4	1979-1989	8000	28,133
Eggs*	4.5	1935-1979	.12	.83
Telephone	13.4	1979-2000	.10	1.40
Kleenex*	17.5	1974-1979	.25	.56
Unit	1.0	0-1979	1.00	356,450,080.
Day's wages	13.4	1910-1980	5.00/day	33,257/day
House	7.59	1957-1979	20,000	100,000
Programmed calculator	-47.6	1974-1979	350	50
Movie*	6.9	1946-1979	.50	4.50
Vicuña	23.0	1979-1989	29,500	233,815
Homicides, Los Angeles	31.2	1978-1988	279/year	4216/year

Items marked (*) show the inflation rate calculated from known price values and years.

The rate of increase of homicides is from the Los Angeles Times, July 11, 1979.

The rate of growth for vicuña is from TIME, July 16, 1979.

The bracketed item was calculated backwards. For a car now costing $8000, at an inflation rate of 7.5%, what would it have cost 33 years ago?

If we had had a steady inflation rate of one percent during the Christian era, then one unit of money at the time of Christ would be some 350,000,000 units now.

Try that the other way around. If an average salary today is, say, $15,000, and an inflation rate of 13.4% continues for the rest of the century, then the average salary in the year 2000 will be around $210,000.

There are at least three powerful forces at work that tend to encourage a high inflation rate:

1. Borrowed money can be paid back with cheaper money. This applies not only to individuals, but to governmental units. If the ratio of the inflation rate to the interest rate on the loan is high enough, it is apparently prudent to borrow. The word is "apparently" inasmuch as the lenders of money can also read, are also subject to inflation, and are not generally stupid. If it actually pays to borrow, who will lend? Nevertheless, a high inflation rate certainly operates to lower the cost of borrowing, and governments are our biggest borrowers.

2. Inflation automatically increases government income without the trauma of raising tax rates. As wages inflate, people move into higher tax brackets, which increases the state's take painlessly. It is somewhat analogous to the situation at gambling casinos, where the odds are correct when you lose, and the casino takes its cut when you win--and when you win, you are much less critical of the loss.

3. Shortly after World War II, television became popular, worldwide. For the first time in history, everyone could see the good things of life close up, in real-time (that is, live, as opposed to still pictures or movies) and, eventually, in color. This was a powerful stimulus for having more people want more things. When the total quantity of goodies is limited and the other inflationary factors are operating as well, all prices must head up in the face of widespread demand.

Seemingly the only ones who are hurt by inflation are those whose income is fixed, or for whom the rate of increase is significantly below the rate of inflation. If all prices go up by 10%, but my income goes up by 11%, then I can be remarkably calm about inflation. But when prices go up by 10% (with a few outstanding items zooming up 20%) and my income remains constant, then the situation is painful.

Interest rate, %	Product of the interest rate (%) and the number of years for money to double				
4	70.6920	70.0056	69.7758	69.6607	69.4533
5	71.0335	70.1776	69.8907	69.7470	69.4879
6	71.3740	70.3493	70.0056	69.8333	69.5225
7	71.7134	70.5208	70.1203	69.9195	69.5570
8	72.0517	70.6920	70.2348	70.0056	69.5916
9	72.3891	70.8629	70.3493	70.0916	69.6262
10	72.7254	71.0335	70.4637	70.1776	69.6607
11	73.0607	71.2039	70.5779	70.2635	69.6953
12	73.3951	71.3740	70.6920	70.3493	69.7298
13	73.7284	71.5438	70.8059	70.4351	69.7643
	1	2	3	4	10
	(Number of times per year that interest is compounded)				

Variations from "The Rule of 72"

It seems clear, just from examining the compound interest calculations, that although there was inflation before World War II, it must have been at a very low level, say 1% or so. Even the experts do not agree on just what happened to boost the rate, worldwide, to the heights we struggle with today. Some of the possible factors are:

1. A change in attitude toward salary raises. Our fathers got raises if and when they deserved them; that is, when they were worth more to the employer. Somewhere along the line, it became the fashion to receive an annual raise just for staying alive for a year, regardless of any value to the employer. This factor has, of course, snowballed; as the rate of inflation increases, the annual "cost of living" raise increases, too.

2. A change in attitude toward equality, of an order that Norman Thomas and Karl Marx never dreamed of. At one time, there was a hierarchy of salary levels, in rough correlation to the prestige level of the job. Thus, vocations could be put into an ordering, something like:

Chief of state
Brain surgeon
Mathematician
Teacher
Mechanic
Bus driver
Trash collector
Street sweeper

Somewhere, things got mixed up, tending to invert the priorities of such a list. As each group jockies for a better position in society, the other groups jockey harder, and the race is on. The race can only send prices up; no group has ever argued for lower salaries.

Inflation is a world-wide disease. Its causes should be traced carefully, in order to understand the phenomenon. The only known cure (perhaps palliative is a better word) is massive governmental controls, which are usually circumvented and which please few people even when they are enforced.

Most articles on inflation conclude with the writer's pet theory on how to cure it. These range from "Let's all work together" to "Start stocking ammunition to use in the inevitable bargaining and bartering."

I have no solutions, weird or otherwise. I can only outline the mathematics involved, and present the pertinent formulas.

The effect of inflation (that is, ordinary compound interest):

$$(1 + R)^{X} = Q$$

where R is the rate of interest, X is the number of time periods (usually in years) and Q shows what one unit of money will amount to. Thus, for 13.4% inflation (annual rate) for ten years, one unit of money will amount to:

$$(1.134)^{10} = 3.51666.$$

The effect, in purchasing power, of that inflation is given by 1/Q. Thus, for the conditions above, the value of $1 today, ten years from now, will be about 28¢.

If the rate of inflation is unknown, but the ratio of prices is known, together with the time span, the formula is:

$$(1 + R)^{Y} = Q$$

where R is the rate, Y is the number of years, and Q is the ratio of prices. We want to solve for R:

$$R = \text{antilog}(\ (\log Q)/Y\ ) - 1$$

For example, the bulk mailing rate went from 5.6¢ to 8.4¢ from 1973 to 1978. The ratio of the prices, Q, is 1.5, and Y = 5. R solves for .08447, or 8.45%. As was pointed out before, the arithmetic fails to tell the whole inflationary story. At the 5.6¢ rate, the postal people did all the work. At the 8.4¢ rate, the mailer does most of the work--sorting, banding, marking, and bagging--under strict and stringent rules.

Fred Gruenberger

K-Column Fibonacci

The accompanying diagram illustrates the problem we have called K-column Fibonacci. It first appeared as Contest Number 9 in our issue 40 and the winning solution, by Sam Wagner, appeared in issue 46.

The problem has two parameters:

K = the number of columns

M = the arithmetic modulus

Thus, the illustration has K = 4, M = 10. The four columns are each initialized to one, as is the first element of the second row. From then on, each new element is the sum of two previous elements, as shown by the boxes in the diagram, reduced modulo M. What is sought is the number of lines, N, needed for the pattern to repeat. For the illustrative case, N = 42. Some other examples, all with K = 1, are given.

For the case K = 1, M = 10, the problem is simply that of the cycle of repetition of the units digit of the Fibonacci sequence (and hence the name), which is well known to be 60. In an accompanying table, we can now present results for a wide range of values of K and M. Most of these are the result of direct computation, using the BASIC program shown.

Other results are derived. Mr. Wagner showed that if we use the notation:

$P_K(M)$ = the number of lines in the repetitive cycle for K columns, modulo M;

then $$P_K(MN) = \text{LCM}\left[P_K(M), P_K(N)\right]$$

LCM is the least common multiple, given by:

$$\text{LCM}(a,b) = \frac{a \cdot b}{\text{GCD}(a,b)}$$

(if a and b were denominators of fractions, the LCM would be called the lowest common denominator.)

1	1	1	1
1	2	3	4
5	6	8	1
5	0	6	4
5	0	0	6
0	5	5	5
1	1	6	1
6	7	8	4
5	1	8	6
0	5	6	4
0	0	5	1
5	5	5	0
1	6	1	6
6	7	3	4
0	6	3	6
0	0	6	9
5	5	5	1
0	5	0	5
6	6	1	1
6	2	8	9
0	6	8	6
5	5	1	9
5	0	5	6
5	0	0	5
1	6	6	6
1	2	8	4
0	1	3	1
5	5	6	9
0	5	0	6
5	5	0	0
6	1	6	6
6	2	3	9
5	1	3	6
5	0	1	4
0	5	5	6
0	0	5	0
6	6	6	1
1	7	3	9
0	1	8	1
0	0	1	9
0	0	0	1
0	0	0	0
1	1	1	1

Four columns, modulo 10

```
1   DIM X(15)
2   DIM Y(15)
10  K = 2
15  L = K - 1
20  M = 10
50  FØR I = 1 TØ K
60  X(I) = 1
70  NEXT I
80  Y(1) = 1
90  N = 0

100 FØR I = 1 TØ L
110 Y(I+1) = (X(I)+Y(I)) MØD M
120 NEXT I
130 N = N+1

200 FØR I = 1 TØ K
210 IF Y(I)<>1 THEN 235
220 NEXT I
225 IF X(K) = 0 THEN 300
235 B = (X(K)+Y(K)) MØD M
240 FØR I = 1 TØ K
250 X(I) = Y(I)
255 NEXT I
256 Y(1) = B
260 GØTØ 100

300 PRINT K, M, N
310 K = K + 1
320 GØTØ 15
```

A possible program in BASIC to calculate the cycle of repetition for K columns, modulo M. Note that the letter oh is slashed here, uniformly.

The statements at 130 and 90 will have to be altered if your BASIC does not count far enough.

$$P_4(10) = LCM\left[P_4(2), P_4(5)\right]$$
$$= LCM\left[21, 6\right]$$
$$= 42$$

$$P_5(42) = LCM\left[P_5(2), P_5(3), P_5(5)\right]$$
$$= LCM\left[63,\ 728,\ 3124\right]$$
$$= 5117112$$

In many cases, the tabular results were derived and also calculated directly as a check.

One of the interesting aspects of the problem is this: there is no a priori reason to believe that any solution exists. Conceivably, any given pattern could short cycle at any time; that is, it could repeat some other pattern than the starting pattern, and this would be difficult to detect. Worse, the pattern might not repeat at all, in the same way that the 10 digits in the decimal expansion of the square root of 2 go on endlessly without repeating.

If further work is to be done on this problem (and it lends itself beautifully to machine language implementation on micro computers), the fundamental calculations should be on values of M that are prime.

1	1	1	1	1	1
1	1	1	1	1	1
0	2	2	2	2	2
1	0	3	3	3	3
1	2	0	5	5	5
0	2	3	1	8	8
1	1	3	6	2	0
1	0	1	0	10	8
0	1	4	6	1	8
	1	0	6	0	3
	2	4	5	1	11
		4	4	1	1
		3	2	2	12
		2	6	3	0
		0	1		12
		2	0		12
		2	1		11
		4	1		10
		1	2		8
		0	3		5
		1	5		0
		1			5
		2			5
					10
					2
					12
					1
					0
					1
					1
					2
					3
					5

One-column Fibonacci, for various moduli.

K = 1 in each case

M = 2 **N** = 3

M = 3 **N** = 8

M = 5 **N** = 20

M = 7 **N** = 16

M = 11 **N** = 10

M = 13 **N** = 28

The vinculum beside each column encloses one complete cycle of repetition.

K	M = 2	M = 3	M = 5	M = 6	M = 7	M = 8	M = 10
2	7	4	31	28	57	14	217
3	5	80	104	80	114	20	520
4	21	39	6	273	84	21	42
5	63	728	3124	46228	480	252	196812
6	127	364	19531	28314	8	254	2480437
7	9	3146	2232	730	1921600	36	2232
8	73	10	97655	1457960	2241867	73	7128815
9	889	1640	1953124	6702276	560200	3556	1736327236
10	1533	13116	243672		2241867	3066	6702276
11	3255	6560			117648	13020	203450520
12	2635	820				2635	
13	11811	4782968				47244	
14	4681	2391484				9362	
15	17	515944				68	
16	273	5157867				273	

Some solutions for the K-Column Fibonacci problem. K, the number of columns, is given along the left. Various values of M, the modulus, are along the top. The table entries are the cycle of repetition.

K	M = 11	M = 13	M = 14	M = 15	M = 17	M = 22
2	30	84	399	124	144	210
3	1330	732	570	1040	32	1330
4	30	183	84	78	72	210
5	118104	371292	10080	568568	88416	354312
6	885775	866348	1016	7109284	4273440	
7		19422160		3510936	83520	
8				195310		
9				800780840		
10				266333496		

K	M = 30	M = 32	M = 42
2	868	56	1596
3	1040	80	4560
4	546	84	1092
5	5117112	1008	131040
6		1016	92456
7		144	
8		146	
9		14224	
10		12264	
11		52080	

The theory tells us that the entry for K = 6, M = 30 should be the LCM(127, 364, 19531) = 902879068.